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Journal of Sound and Vibration 304 (2007) 272–283
JOURNAL OF SOUND AND VIBRATION
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Vertical vibrations of rigid circular disks under ground-transmitted high-frequency waves

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Received 5 June 2006; received in revised form 25 February 2007; accepted 3 March 2007

Abstract

The vertical vibrations of rigid circular disks on elastic and viscoelastic stratums subjected to high-frequency vertical excitations are studied. The responses of rigid disks subjected to ground-transmitted harmonic and transient waves are discussed. It is found that the existence of the stratums has damping effect on the responses of the disks. The damping effect is dependent upon the Poisson's ratio of the stratum, its depth and material damping as well as the excitation frequency and the mass of the disk.

1. Introduction

In soil dynamics and foundation engineering, the dynamic interaction of a structure with ground is an important subject, and it has received considerable attention [1]. A topic of great interest in understanding the interaction between soil and structure is the dynamic force–displacement relationship of a rigid body attached to a half-space or a layered half-space. Such relationships, when expressed as influence functions or interaction coefficients, reduce the problem of finding the dynamic response of surface structures to a set of algebraic equations. They can be used as a basis for the analysis of the response of surface structures to dynamic loadings, in particular, underground explosive loadings, seismic excitations and machine vibrations. Of particular importance is the force–displacement relationship of a harmonically vibrating rigid plate. Once this relationship has been established, the harmonic or transient response of any linear foundation may be evaluated by standard procedures.

For an elastic half-space, a large amount of research work has been done on the subject beginning with the work of Lamb [2]. A few analytical techniques such as integral transform, Green's functions, etc., as well as boundary element and other numerical methods have been presented to deal with various aspects of the topic.

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2007.03.004

Collins [3] studied the torsional oscillations of a rigid disk supported by a semi-infinite solid. Awojobi and Grootenhuis [4], Zakorko and Rostovtsev [5], Robertson [6] and Gladwell [7] analyzed the vertical, tangential and/or rocking oscillations of a smooth disk in contact with a half-space. On the other hand, Karasudhi et al. [8] determined the force–displacement relationships due to harmonic oscillations of a smooth rectangular footing at the surface of a half-space. Luco and Westman [9] discussed the vertical vibrations of a rigid body on an elastic half-space. Veletsos and Tang [10] investigated the vibrations of a rigid annular body attached to a half-space.

As known, the elastic stratum is a more realistic representation of the actual medium in practice. However, all the works mentioned above are restricted to the case of a rigid body on a homogeneous elastic half-space. To remove such limitation by representing the ground as an elastic medium consisting of a layer of constant thickness supported on an elastic half-space Awojobi [11] dealt with the torsional oscillations of a rigid circular body and discussed the practical need to study vibrations of rigid bodies on a stratum rather than the conventional half-space. For the same case of torsional oscillations, Gladwell [12] obtained approximate low-frequency solutions. For the vertical vibrations of a rigid circular body and the rocking of a long rigid rectangular body on an infinitely wide elastic stratum, Awojobi [13] formulated the mixed boundary-value problems in terms of dual integral equations and obtained approximate solutions of these equations in low-frequency range. For structures of high inertia ratios, the low-frequency solutions would be adequate in most cases if interest in the problem were limited to the prediction of resonant frequency. However, the low-frequency solutions often become invalid in high-frequency range that may be quite apart from a consideration of resonance. Accordingly, Awojobi [14] delineated the high-frequency torsional vibrations of a rigid circular body on an elastic stratum and vertical vibrations on a half-space under harmonic loading. The problems were solved without assuming the form of the unknown dynamic stress distribution under the rigid bodies, and the exact formulations of the problems in terms of dual integral equations were solved by finding a dominant approximation to these equations at very high frequency.

It is noted that Awojobi [14] only gave the solution for the elastic half-space in the case of vertical high-frequency vibration under harmonic loading. Thus, in this paper, the vertical vibrations of rigid circular disks on elastic and viscoelastic stratums under high-frequency vertical excitations are studied. The responses of rigid disks subjected to vertically incident harmonic and transient waves are discussed, respectively.

2. Response of disk on elastic stratum

2.1. Governing dual integral equations and method of solution

Consider a rigid circular disk of radius a subjected to vertical vibration with the angular frequency of harmonic excitation ω at the surface of an elastic stratum (see Fig. 1). The depth of the stratum is denoted by h, and its Poisson's ratio and shear modulus are represented by v and G, respectively. Then the velocities of the

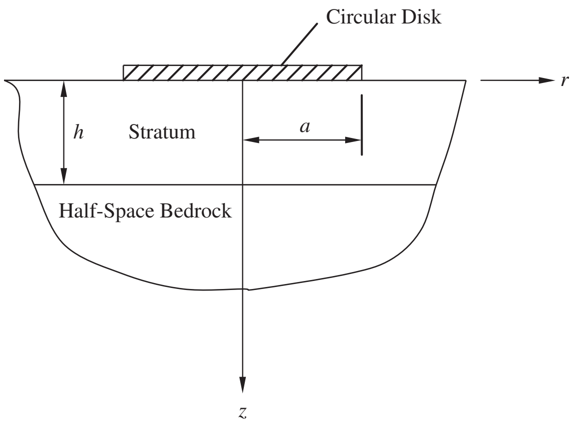

Fig. 1. Geometry of the problem under consideration.

compressional and shear waves V_c and V_s can be expressed as

$$V_c^2 = \frac{2(1-v)G}{\rho(1-2v)}, \tag{1}$$

$$V_s^2 = \frac{G}{\rho}, \tag{2}$$

where ρ denotes the density of the stratum. In addition, the zero-order Hankel transform of the unknown stress distribution $\sigma(r)$ immediately under the rigid disk can be obtained by

$$\tau(p) = \overline{\sigma(r)} = \int_0^\infty \sigma(r) r J_0(pr)\,\mathrm{d}r,$$

where p is the transform parameter.

Introducing the non-dimensional quantities $\eta = pa$, $\eta_1 = \omega a/V_c$, $\eta_2 = \omega a/V_s$, $R = r/a$ and $H = h/a$, we have the following governing dual integral equations [13]:

$$\frac{\eta_2^2}{G}\int_0^\infty \frac{\alpha_1 F(\eta) J_0(\eta R)\,\mathrm{d}\eta}{(2\eta^2-\eta_2^2)^2\coth(\alpha_1 H) - 4\eta^2\alpha_1\alpha_2\coth(\alpha_2 H)} = u_z^0 \quad (0<R<1), \tag{3}$$

$$\int_0^\infty F(\eta) J_0(\eta R)\,\mathrm{d}\eta = 0 \quad (R>1), \tag{4}$$

where u_z^0 denotes the vertical displacement of the rigid disk, and

$$F(\eta) = p\tau(p),$$

$$\alpha_1 = \sqrt{\eta^2-\eta_1^2},$$

$$\alpha_2 = \sqrt{\eta^2-\eta_2^2}.$$

It is noted that no exact solution of the pair of Eqs. (3) and (4) is, at present, available. It is therefore necessary to seek an approximate solution for the pair of equations. It is found that the function $y = x/(1+x^2)^{1/2}$ is adequately close to the hyperbolic tangent function and the two functions and their derivatives are identical at the origin and at infinity [11]. Consequently, Eqs. (3) and (4) become

$$\int_0^\infty \frac{\alpha_1^2}{\psi} F(\eta) J_0(\eta R)\,\mathrm{d}\eta = \frac{G u_z^0}{\eta_2^2} \quad (0<R<1), \tag{5}$$

$$\int_0^\infty F(\eta) J_0(\eta R)\,\mathrm{d}\eta = 0 \quad (R>1), \tag{6}$$

where

$$\psi = (2\eta-\eta_2^2)^2\sqrt{\eta^2-\eta_{1e}^2} - 4\eta^2(\eta^2-\eta_1^2)\sqrt{\eta^2-\eta_{2e}^2},$$

with

$$\eta_{1e} = \sqrt{\eta_1^2 - 1/H^2}, \quad \eta_{2e} = \sqrt{\eta_2^2 - 1/H^2}.$$

As known from Awojobi [14], the frequency factor $\eta_2 = \omega a/V_s$ is a fundamental parameter in the vibrations of rigid bodies on an elastic medium. It is noted that $\eta_2/\eta_1 = V_c/V_s$ and normally $\eta_2 > \eta_1$. In general, the vibration is in high-frequency range when the value of η_2 is greater than unity. At high-frequency factors, the contribution to the integral in Eq. (5) comes from small values of η. Hence, for α_1, α_2 and ψ we have the following approximate expressions in the range $\eta < \eta_1$

$$\alpha_1 \approx \mathrm{i}\eta_2 \sqrt{\frac{1-2v}{2(1-v)}} \quad (0<\eta<\eta_1<\eta_2), \tag{7}$$

$$\alpha_2 \approx \mathrm{i}\eta_2 \quad (0<\eta<\eta_1<\eta_2), \tag{8}$$

$$\psi \approx \mathrm{i}\eta_2^4\eta_{1e} \quad (0<\eta<\eta_1<\eta_2), \tag{9}$$

where the imaginary number $\mathrm{i}=\sqrt{-1}$. Substitution of the above approximate expressions into the governing dual integral equations leads to

$$\int_0^\infty F_1(\eta)J_0(\eta R)\,\mathrm{d}\eta = \mathrm{i}\,\frac{2(v-1)Gu_z^0\eta_{1e}}{(1-2v)} \quad (0<R<1), \tag{10}$$

$$\int_0^\infty F_1(\eta)J_0(\eta R)\,\mathrm{d}\eta = 0 \quad (R>1), \tag{11}$$

with

$$F_1(\eta) = \mathrm{i}\,\frac{2(v-1)}{(1-2v)}G\eta_{1e}u_z^0 J_1(\eta), \tag{12}$$

where $F_1(\eta)$ corresponds to the approximation solution of $F(\eta)$ and in which all contributions to the first integrals from the range $\eta>\eta_1$ have been neglected.

2.2. Force–displacement relationship

To obtain the relationship between the external force and the displacement of the rigid disk, the dynamic stress distribution under the rigid disk should be determined. Using Hankel's inversion theorem, we have

$$\sigma(r) = \int_0^\infty \tau(p)pJ_0(pr)\,\mathrm{d}p.$$

It follows from the above equation that

$$\begin{aligned}\sigma(R) &= \frac{1}{a}\int_0^\infty F(\eta)J_0(\eta R)\,\mathrm{d}\eta \\ &\approx \frac{\mathrm{i}2G(v-1)\eta_{1e}u_z^0}{a(1-2v)}\int_0^\infty J_1(\eta)J_0(\eta R)\,\mathrm{d}\eta,\end{aligned} \tag{13}$$

where use is made of the following approximate solution:

$$F(\eta) \approx F_1(\eta). \tag{14}$$

It follows from Eq. (13) that

$$\sigma(R) \approx \mathrm{i}\eta_{1e}\frac{2G(v-1)u_z^0}{a(1-2v)} \quad (0<R<1), \tag{15}$$

$$\sigma(R) = 0 \quad (R>1). \tag{16}$$

We now consider the vibrations of the rigid disk under an harmonic force $F\mathrm{e}^{\mathrm{i}\omega t}$. By the use of the impedance method, we have

$$\int_0^1 \sigma(R)a^2 2\pi R\,\mathrm{d}R - m\omega^2 u_z^0 + F = 0, \tag{17}$$

which is the equation of vertical vibrations under the excitation for the disk of mass m. Substituting for $\sigma(R)$ from Eq. (15) into Eq. (17) and introducing the non-dimensional mass of the disk $M = m/\rho a^3$, we obtain

$$F = Ku_z^0, \tag{18}$$

where K is the complex-valued vertical impedance function. The impedance function is usually written in the form

$$K = Ga(P + \mathrm{i}\eta_2 Q), \tag{19}$$

where P and Q are normalized stiffness and damping coefficients, respectively, and they can be expressed as

$$P = M\eta_2^2, \tag{20}$$

$$Q = \frac{2(1-v)\pi\eta_{1e}}{(1-2v)\eta_2}. \tag{21}$$

2.3. Response to wave excitations

To determine the response of the disk subjected to elastic wave excitation, care must be exercised in determining the appropriate input ground motion. It is known that the effective foundation input motion u^* is the response of a massless foundation with the same shape as the actual foundation when subjected to the elastic wave excitation in the absence of other external forces. If the foundation is under a free-field ground motion characterized by the amplitude $u_g e^{\mathrm{i}\omega t}$ on the surface of the ground, we have

$$u^* = Su_g, \tag{22}$$

where S stands for the input motion factor representing the kinematic interaction effects. It should be pointed out that numerical values for the factor S for various foundation geometries and types of excitation have been given by Day [15], Dominguez [16], Wong and Luco [17] and Mita and Luco [18]. In the particular case of a surface foundation (no embedment) subjected to vertically incident waves, $S = 1$ and $u^* = u_g$. In all other cases, $S \neq 1$ and $u^* \neq u_g$.

For the case of vertically incident harmonic wave excitation, Eq. (18) can be rewritten as

$$Ku_z^0 - m\omega^2 u_z^* = 0, \tag{23}$$

where u_z^* denotes the effective ground input motion in the vertical direction. It follows that

$$\frac{u_z^0}{u_z^*} = \frac{m\omega^2}{K}. \tag{24}$$

Then the magnitude of the amplitude ratio can be expressed as

$$\chi = \left|\frac{u_z^0}{u_z^*}\right| = \frac{M\eta_2^2}{\sqrt{\beta(\pi\eta_2^2 - \beta/H^2) + M^2\eta_2^4}}, \tag{25}$$

where

$$\beta = \frac{2(1-v)\pi}{1-2v}.$$

When the Poisson's ratio of the stratum $v = 0.3$ and the non-dimensional mass of the disk $M = 2.0$, the effect of the depth of the stratum on the response of the disk is obtained from Eq. (25) and shown in Fig. 2. It is seen that the difference between the values of the two displacement amplitudes becomes less pronounced with increasing the exciting frequency and decreasing the depth of the stratum. It can also be found that the sensitiveness of the amplitude ratio to the depth of the stratum decreases with the increase of the exciting frequency. In addition, from Fig. 2 it can be obviously concluded that the existence of the stratum has the radiation damping effect on the response of the disk, and the damping effect gets more significant with the increase of the depth of the stratum and the decrease of the exciting frequency.

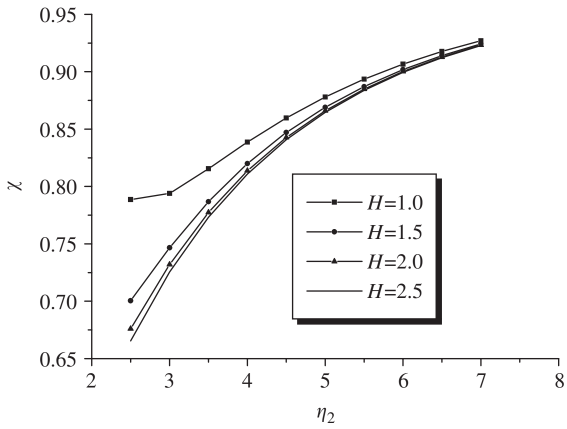

Fig. 2. Effect of the depth of stratum on the response of disk.

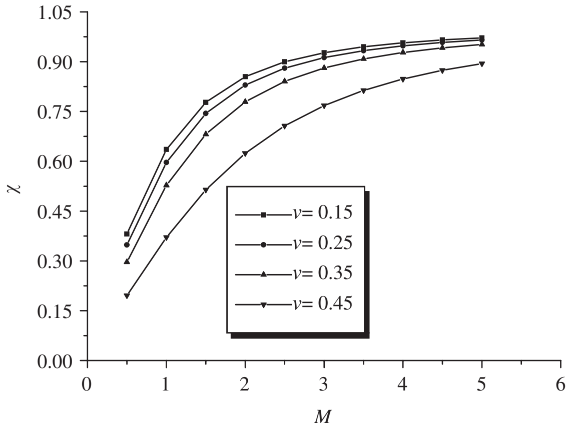

Fig. 3. Influence of the Poisson's ratio of stratum on the response of disk.

When the non-dimensional depth of the stratum $H = 3.0$ and the exciting frequency $\eta_2 = 4.0$, the variation of the magnitude of the amplitude ratio with the Poisson's ratio of the media and the mass of the disk is illustrated in Fig. 3. As shown, if the radius of the disk keeps unchanged, the distinction between the values of the two displacement amplitudes gets more visible with the increase of the Poisson's ratio of the stratum and the diminishment of the mass of the disk. This implies that the radiation damping effect of the stratum on the response of the disk becomes more pronounced when the Poisson's ratio of the stratum increases and the mass of the disk decreases.

In the following, we study the problem regarding the response of a rigid disk on an elastic stratum subjected to a ground-transmitted transient excitation.

In a typical dynamic analysis, the basic mathematical model adopted is a lumped mass with a spring and dashpot. The response of the mass is dependent on the property of the stratum reaction, which may be modeled by the spring and the dashpot. When subjected to a ground-transmitted excitation, the equation of motion for this rigid disk in the vertical direction can be obtained by [19]

$$m\ddot{u}_z^0 + c\dot{u}_z^0 + ku_z^0 + m\ddot{u}_z(t) = 0, \tag{26}$$

where $\ddot{u}_z(t)$ is the absolute ground acceleration time history in the vertical direction measured at the location of the future disk foundation, and parameter k and c are the true stiffness constant and damping constant, which is defined by

$$k = GaP = GaM\eta_2^2, \tag{27}$$

$$c = \frac{Ga\eta_2}{\omega}Q = \frac{2(1-\nu)G}{(1-2\nu)}\frac{a^2\pi\eta_{1e}}{V_s\eta_2}. \tag{28}$$

In the case of the transient excitation, a Fourier analysis can be used to obtain the response of the disk in the frequency domain. In this case, the excitation can be simulated by the sum of the harmonic components, which are obtained by the use of the discrete Fourier transform (DFT).

Under the action of the nth harmonic component of the excitation in the vertical direction, the response of the disk is governed by

$$m\ddot{u}_z^{0(n)} + c\dot{u}_z^{0(n)} + ku_z^{0(n)} = W_n e^{i\omega_n t}, \tag{29}$$

where W_n and ω_n are the amplitude and frequency of that harmonic component. Furthermore, we have [19]

$$u_z^{0(n)}(t) = H_n W_n e^{i\omega_n t}, \tag{30}$$

where the transfer function H_n is obtained by

$$H_n = \frac{1}{k}\frac{1}{1-\gamma^2+i2\tau\gamma} = |H_n|e^{i\phi_n}, \tag{31}$$

where $|H_n|$ denotes the modulus of the transfer function, and

$$\gamma = \frac{\omega_n}{\bar{\omega}}, \tag{32}$$

$$\tau = \frac{c}{2\sqrt{km}}, \tag{33}$$

$$\phi_n = \tan^{-1}[-c\omega_n/(k-m\omega_n^2)] \tag{34}$$

with

$$\bar{\omega} = \sqrt{\frac{k}{m}}.$$

On the basis of the principle of superposition, the total response is given by

$$u_z^0(t) = \sum u_z^{0(n)}(t). \tag{35}$$

A case of interest is the response of a disk subjected to blast-induced ground shock. It is assumed that the radius of the disk is 1.0 m, and its mass is 4000.0 kg. The inertial force due to the blast-induced ground motion at free ground surface is adopted as the dynamic excitation force. The Fourier amplitude spectrum of a typical inertial force due to blast-induced ground shock in the vertical direction is shown in Fig. 4.

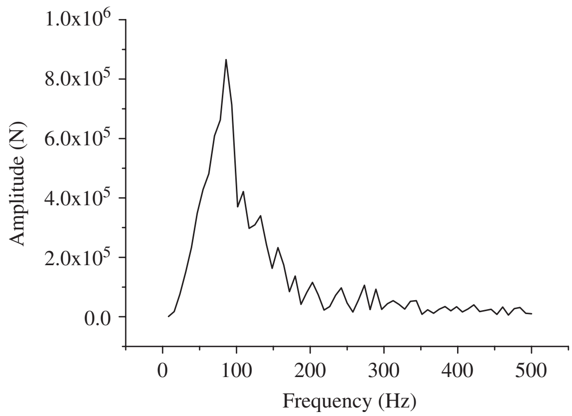

Fig. 4. Fourier amplitude spectrum of a typical inertial force due to ground shock.

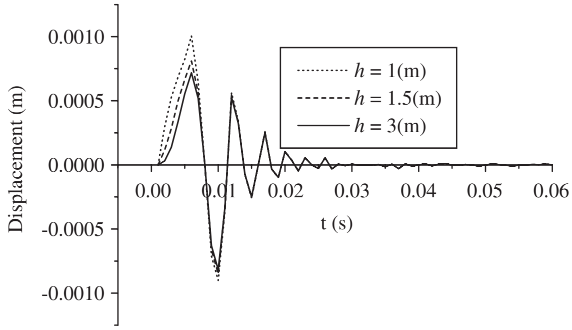

Fig. 5. Influence of the depth of stratum on the displacement of disk under blast-induced ground shock.

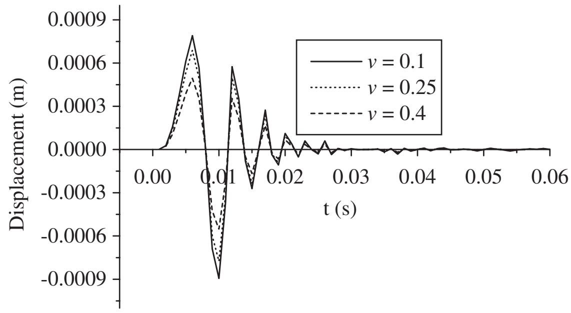

Fig. 6. Effect of the Poisson's ratio of stratum on the displacement of disk under blast-induced ground shock.

When the Poisson's ratio of the stratum $v = 0.2$, the density of the stratum $\rho = 2500\,\mathrm{kg/m^3}$, and its shear modulus $G = 40\,\mathrm{MPa}$, the influence of the depth of the stratum on the displacement of the disk under blast loading is obtained, which is shown in Fig. 5. As can be seen, the peak displacement for the disk is reduced with the increase of the depth of the stratum, which implies that the radiation damping effect of the stratum becomes more significant with the depth of the stratum increased.

For a particular case with the depth of the stratum $h = 4\,\mathrm{m}$, the density of the stratum $\rho = 2500\,\mathrm{kg/m^3}$, and its shear modulus $G = 40\,\mathrm{MPa}$, the effect of the Poisson's ratio of the stratum on the displacement of the disk under blast loading is indicated in Fig. 6. As shown, the peak displacement for the disk decreases with increasing the value of the Poisson's ratio of the stratum. Thus, it can be concluded that the radiation damping effect of the stratum gets more noticeable when the value of the Poisson's ratio of the stratum increases.

3. Response of disk on viscoelastic stratum

There exist two mechanisms of energy dissipation in foundation dynamics, namely wave radiation and material damping. Elastic continuum models of a medium such as soil under the disk capture only the radiation effect. However, the effect of material damping may become rather important in the case of the supporting medium subjected to high-intensity excitations. To incorporate material damping, we herein idealize the stratum as a linear viscoelastic solid, and the constant hysteretic model of viscoelastic action is considered.

By application of the correspondence principle [20], the impedance function K^v for the case of a disk on the viscoelastic stratum can be determined from Eq. (19) simply by replacing the real-valued shear modulus G by a complex modulus $\tilde{G}$. It should be mentioned that herein an assumption has been made that Poisson's ratio for the viscoelastic material is the same as that for the material in the corresponding elastic problem. When $v = 1/2$, no approximation exists in this assumption. The inaccuracy that may be involved for all other values of v of practical interest is expected to be negligible for most engineering purposes.

The complex modulus $\tilde{G}$ is defined by

$$\tilde{G} = G\left(1 + \mathrm{i}\frac{\omega G'}{G}\right), \tag{36}$$

where G' is the shear modulus of viscosity, and it is defined by

$$G' = \frac{G \tan \delta}{\omega}, \tag{37}$$

in which $\tan\delta$ denotes the coefficient of friction with δ standing for a sort of angle of mobilized internal friction for the soil.

It should be noted that the value of $\tan\delta$ is normally less than 0.05 at small strains, whereas for the much larger strains associated with high-intensity motions, its value may be as high as 0.3 or 0.4. The maximum possible upper limit of δ is the angle of repose of sand slopes, around 35°, which implies that $\tan\delta$ can never exceed about 0.7.

Upon making the indicated substitutions, Eq. (19) can be written as

$$K^v = \tilde{G}a(\tilde{P} + \mathrm{i}\tilde{\eta}_2\tilde{Q}), \tag{38}$$

where superscript v refers to the viscoelastic problem, and

$$\tilde{P} = M\tilde{\eta}_2^2, \tag{39}$$

$$\tilde{Q} = \frac{2(1-v)\pi\tilde{\eta}_{1e}}{(1-2v)\tilde{\eta}_2} \tag{40}$$

with

$$\tilde{\eta}_2^2 = \frac{\rho\omega^2 a^2}{\tilde{G}}, \tag{41}$$

$$\tilde{\eta}_{1e}^2 = \frac{\rho(1-2v)\omega^2 a^2}{2(1-v)\tilde{G}}. \tag{42}$$

Separating the real and imaginary parts, Eq. (38) can also be written in the form

$$K^v = Ga(P^v + \mathrm{i}\eta_2 Q^v), \tag{43}$$

where P^v and Q^v are real-valued functions which can be, respectively, expressed as

$$P^v = A_1 - A_5\sqrt{\frac{\Delta - A_3}{2}} - \left(A_2 + A_5\sqrt{\frac{\Delta + A_3}{2}}\right)\tan\delta, \tag{44}$$

$$Q^v = \frac{1}{\eta_2}\left[A_2 + A_5\sqrt{\frac{\Delta + A_3}{2}} + \left(A_1 - A_5\sqrt{\frac{\Delta - A_3}{2}}\right)\tan\delta\right], \tag{45}$$

in which

$$A_1 = \frac{M\eta_2^2}{1+\tan^2\delta},$$

$$A_2 = \frac{-M\eta_2^2\tan\delta}{1+\tan^2\delta},$$

$$A_3 = \frac{H^2\eta_2^2(1-2v) - 2(1-v)(1+\tan^2\delta)}{2(1-v)(1+\tan^2\delta)H^2},$$

$$A_4 = -\frac{\eta_2^2(1-2v)\tan\delta}{2(1-v)(1+\tan^2\delta)},$$

$$A_5 = \frac{2(1-v)\pi}{1-2v},$$

$$\Delta = \sqrt{A_3^2 + A_4^2}.$$

Replacing the impedance function K in Eq. (24) with the function K^v, we obtain the amplitude ratio of the deformation of the disk to that of the effective ground input as follows:

$$\chi = \left|\frac{u_z^0}{u_z^*}\right| = \frac{M\eta_2^2}{\sqrt{(P^v)^2 + \eta_2^2(Q^v)^2}}. \tag{46}$$

As an example, assuming the Poisson's ratio of the stratum $v = 0.3$, the non-dimensional depth of the stratum $H = 3.0$, and the non-dimensional mass of the disk $M = 2.0$, the effect of the material damping on the response of the disk is obtained from Eq. (46) and shown in Fig. 7. As can be seen, the ratio χ is reduced with the increase of the material damping. In other words, the damping effect of the stratum gets more pronounced with increasing the material damping.

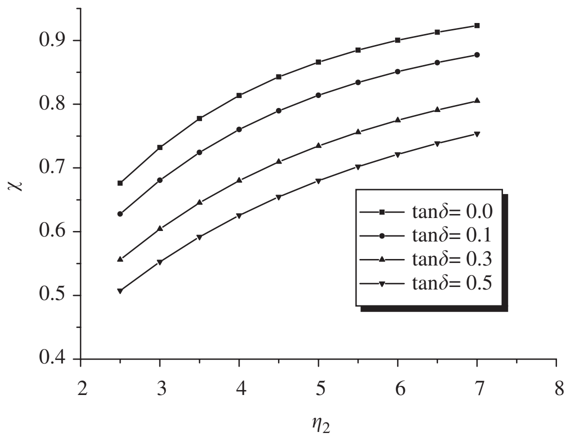

Fig. 7. The effect of the material damping on the response of the disk.

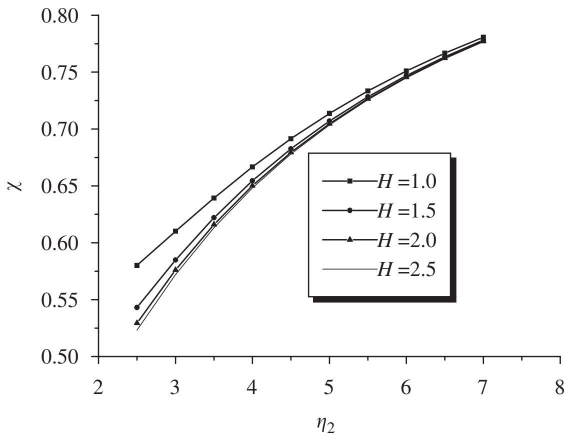

Fig. 8. Influence of the depth of stratum on the response of disk.

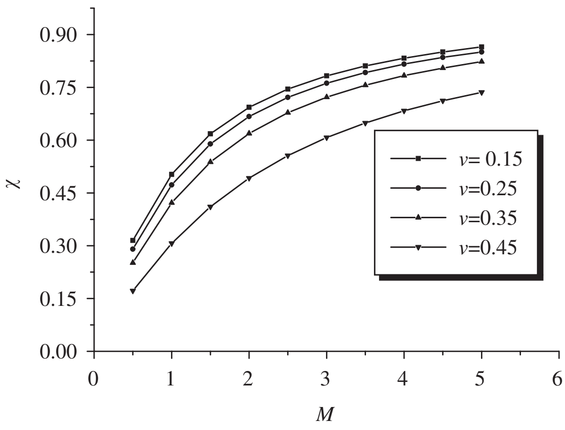

Fig. 9. Influences of the Poisson's ratio of stratum on the response of disk.

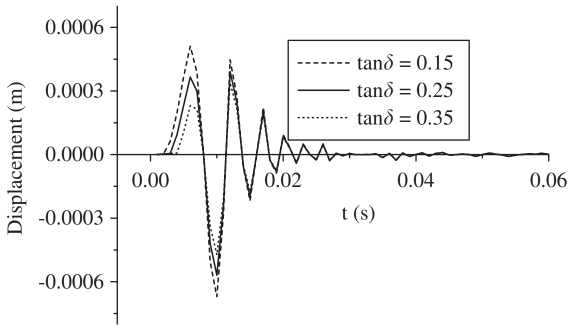

Fig. 10. Effect of material damping of stratum on the response of disk under blast-induced ground shock.

For the case with the Poisson's ratio of the stratum $v = 0.3$, the non-dimensional mass of the disk $M = 2.0$, and $\tan\delta = 0.4$, the influence of the depth of the stratum on the response of the disk is obtained from Eq. (46) and illustrated in Fig. 8. It can be observed that the damping effect becomes more pronounced when the depth of the viscoelastic stratum increases and the excitation frequency decreases. This is consistent with that shown in Fig. 2 for the case of elastic stratum.

When the non-dimensional depth of the stratum $H = 3.0$, the exciting frequency $\eta_2 = 4.0$, and $\tan\delta = 0.4$, the variation of the magnitude of the amplitude ratio with the Poisson's ratio of the media and the mass of the disk is indicated in Fig. 9. As can be seen, the influences of the Poisson's ratio of the stratum and the mass of the disk on the damping effect of viscoelastic stratum are similar to those shown in Fig. 3 for the case of elastic stratum.

In the case of rigid disk on viscoelastic stratum subjected to ground-transmitted transient excitation, the true stiffness constant k^v and damping constant c^v can be, respectively, obtained by

$$k^v = GaP^v, \tag{47}$$

$$c^v = a^2\sqrt{\rho G}Q^v. \tag{48}$$

By the replacement of k and c with k^v and c^v, the viscoelastic solutions can be determined from the corresponding elastic solutions.

In what follows, the response of the disk supported by a viscoelastic stratum under blast-induced ground shock is considered. Both the radius of the disk and its mass are the same as those in elastic case, and the blast loading is also the same. With the depth of the stratum $h = 4\,\text{m}$, the density of the stratum $\rho = 2500\,\text{kg/m}^3$, and its shear modulus $G = 40\,\text{MPa}$, the effect of material damping of the stratum on the response of the disk under blast loading is illustrated in Fig. 10. It can be observed that the peak displacement for the disk decreases with the increase of material damping of the stratum.

4. Conclusions

The vertical vibrations of rigid circular disks on elastic and viscoelastic stratums under high-frequency vertical excitations are investigated. The responses of rigid disks subjected to ground-transmitted harmonic and transient waves are discussed, respectively.

It is concluded that the existence of the stratum has the radiation damping effect on the response of the disk, and the radiation damping effect becomes more significant with the increase of the depth of the stratum and its Poisson's ratio and with the decrease of the excitation frequency and the mass of the disk. On the other hand, the radiation damping effect of the stratum is also dependent on the material damping, and it gets more pronounced with the augmentation of the material damping.

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